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Spectrum Sensing and Throughput Analysis for Full-Duplex Cognitive Radio with Hardware Impairments

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Abstract

In Full-duplex Cognitive Radios, the silent period of the Secondary User (SU) during the Spectrum Sensing can be eliminated by applying the Self-Interference Cancellation (SIC). Due to the channel estimation error and the hardware imperfections (the Phase Noise and the Non-Linear Distortion (NLD)) SIC is not perfectly performed and results in the Residual Self-Interference (RSI) which affects the Spectrum Sensing reliability. In this paper, the effect of RSI on Spectrum Sensing is analytically derived by expressing the detection (p_d) and false alarm (p_{fa}) probabilities under FD in terms of the ones under Half-Duplex (HD) (where SU should remain silent during the Spectrum Sensing period). In addition, an algorithm is proposed to suppress the NLD and improves the Spectrum Sensing performance. Hereinafter, the SU throughput under FD is analysed comparing to HD by deriving the upper and lower bounds to be respected by p_{fa} and p_d respectively in order to make FD beneficial relative to HD.

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1. Introduction

Recently, the Full Duplex (FD) transmission has been introduced in the context of Cognitive Radio (CR) to enhance the Data-Rate of the Secondary (unlicensed) User (SU). In FD systems, SU can simultaneously transmit and sense the channel. In classical Half Duplex (HD) systems, the SU should stop transmitting in order to sense the status of the Primary (licensed) User (PU). Recent advancements in the Self-Interference Cancellation (SIC) make the application of FD in CR possible. Due to many imperfections, a perfect elimination of the self-interference cannot be reached in real world applications [2, 3]. In CR, the SU makes a decision on the PU status using a Test Statistic [4–6]. This Test Statistic depends on the PU signal and the noise. Any residual interference from the SU signal can affect the Test Statistic norm and leads to a wrong decision about the presence of PU.

In wireless systems, the FD is considered as achieved if the Residual Self Interference (RSI) power becomes lower than the noise level. For that an important SIC gain is required (around 110 dB for a typical WiFi system [2]). This gain can be achieved using a passive suppression and an active cancellation. The passive suppression is related to many factors that reduce the Self Interference (SI) such as the transmission direction, the absorption of the metals and the distance between the transmitting antenna, T_x, and the receive antenna, R_x. The active cancellation reduces the Self-Interference (SI) by using a copy of the known transmitted signal. The estimation of channel coefficients becomes an essential factor in the active cancellation process. Any

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error in the channel estimation leads to decreasing the SIC gain.

Experimental results show that hardware imperfections such as the non-linearity of amplifier and the oscillator noise are the main limiting performance factors [2, 7–9]. Therefore, the SIC should also consider the receiver imperfections. The authors of [2] modify their previous method of [10] to estimate the channel and the Non-Linearity Distortion (LND) of the receiver Low -Noise Amplifie (LNA). Their method requires two training symbol periods. During the firs period, the channel coefficients are estimated in the presence of the NLD. The non-linearity of the amplifie is estimated in the second period using the already estimated channel coefficients. It is worth mentioned that the estimation of the NLD parameters in the second phase depends on the one of the channel coefficients done in the firs phase. However the estimation of the channel coefficients in the firs phase can be depending on unknown NLD parameters. To solve the previous dilemma, we propose hereinafter an estimation method of the NLD in such way that the estimation of the channel cannot be affected by the NLD.

The works of [11–16] deal with the application of FD in CR. In [11–13, 15], the RSI is modeled as a linear combination of the SU signal without considering hardware imperfections. In [13, 16] the Energy Detection (ED) is studied in a FD mode and the probability of detection, (p_d), and false alarm, p_{fa}, are found analytically. According to our best knowledge, there is no analytic relationship between the RSI, p_d and p_{fa} for both HD and FD mode.

Further, due to the RSI, p_d and p_{fa} are highly affected. This fact impacts negatively the SU throughput. In some circumstances, the SU throughput under HD can be higher than the one of FD. This can be occurred when the false alarm rate is high. On the other hand, PU transmission is considerably disturbed by the SU activities when the detection rate becomes low. For that reason, upper bound of false alarm rate and lower bound of detection rate are important in order to be abided by SU in FD mode in order to enhance the throughput without increasing the interference rate to PU comparing to HD mode.

This paper deals with the Spectrum Sensing in real world applications. At firs , we analytically determine the impact of the RSI power on the detection process. For that objective, we derive a relation between the RSI power, the probabilities of detection and false alarm under HD and FD modes. Secondly, we analyse the NLD impact on the channel estimation and the Spectrum Sensing Performance. Hereinafter, novel algorithms are proposed to suppress the NLD of LNA.

In addition, the effect of FD mode on the SU throughput is compared to the HD mode. As the throughput is related to p_d and p_{fa}, the upper bound of false alarm rate and the lower bound of the detection rate are derived. These two bounds characterize the required limit in FD in order to enhance the SU throughput comparing to HD without making an additional interference to PU.

The rest of this paper is presented as follows, in section (2), an overview of OFDM receiver is presented by focusing on the circuit imperfections. The Spectrum Sensing hypothesis in Full-Duplex Cognitive Radio is presented in section (3), where the effect of RSI is analysed analytically. In section (4), an algorithm of the NLD mitigation in RF domain is proposed with its corresponding numerical results. Throughput analysis in terms of detection and false alarm probabilities is presented in section (5). At the end, section (6) concludes the work by providing new perspectives.

Throughout this paper, uppercase letters represent frequency-domain signals and lower-case letters represent signals in time-domain.

2. Receiver chain Imperfection Analysis

In this section, we analyse the imperfections in a classical OFDM receiver. By referring to figur (1), which represents a typical Ordinary Chain (OC) of an OFDM receiver block-diagram, the SU signal received at R_x is modelled as:

$$y(t) = h(t) * s(t) \exp(j2\pi f_c t) \tag{1}$$

where $h(t)$ is the channel effect between T_x and R_x and $*$ stands for the convolution operator. After that, $y(t)$ is amplifie using the LNA which introduces a NLD. The output of LNA can be modelled in a general form as a polynomial of odd degrees:

$$y_a(t) = \sum_{i=0}^{\infty} a_{2i+1} y^{2i+1}(t) \tag{2}$$

The firs coefficient a_1, stands for the linear component which represents the amplifie signal. The other coefficients (a_{2i+1}, $i \geq 2$) stand for the non-linear components contributing in NLD. Since the higher component are of negligible power, the NLD of the output of the LNA is limited to the third order polynomial output:

$$y_a(t) = a_1 y(t) + a_3 y^3(t) \tag{3}$$

In digital domain, $y_a(n)$ can be expressed as follows [3, 17]:

$$y_a(n) = a_1 y(n) + a_3 y(n)|y(n)|^2 \tag{4}$$

After the amplific tion process, the received signal is down-converted to the base-band form. As shown in figur (1), the oscillator can introduce a multiplicative noise $\exp\left(j\phi(t)\right)$.

After converting to the base-band, the Analog-to-Digital Converter (ADC) digitizes the received signal and introduces a uniform noise $w_q(n)$, which has a power inversely proportional to the number of used bits.

Consequently, the received time-domain base-band signals at OC is presented as follows:

$$\begin{aligned} y_a(n) &= \left(a_1 y(n) + a_3 y(n)|y(n)|^2\right)\exp\left(j\phi(n)\right) \\ &+ w_q(n) + w(n) \\ &= \Big(a_1 h(n) * s(n) \\ &+ a_3(h(n)*s(n))|h(n)*s(n)|^2\Big)\exp\left(j\phi(n)\right) \\ &+ w_q(n) + w(n) \end{aligned} \tag{5}$$

Where $w(n)$ is an additive zero mean white Gaussian noise. $w(n)$ is the internal noise in the receiver circuit and it is related to the input signal level and to the blocks of the receiver chain.

FFT and CP removal operations will be applied on $y_a(n)$ to obtain $Y_a(m)$. By assuming that $w_q(m)$ is dominated by $w(n)$, $Y_a(m)$ can be presented as follows:

$$Y_a(m) = a_1 H(m)S(m) * B(m) + D(m) + W(m) \tag{6}$$

Where $D(m) = a_3 FFT\left(y(n)|y(n)|^2\right) * B(m)$ is the NLD of the LNA in frequency domain and $B(m) = FFT\left(\exp\left(j\phi(n)\right)\right)$.

In frequency domain, the channel estimation is done in order to perform SIC, as well as the circuit imperfection mitigation. The frequency domain SU signal $S(m)$, contributing in the channel estimation and the imperfection mitigation is coming from an Auxiliary Chain (AC), which is designed in a way to mitigate the imperfections and to help estimate the channel.

After the SIC and the circuit imperfections mitigation, the obtained signal, $\hat{Y}(m)$, can be presented as follows:

$$\hat{Y}(m) = \xi(m) + W(m) + W_q(m) + \eta X(m) \tag{7}$$

Where $\xi(m)$ is the RSI and define as:

$$\xi(m) = (H(m) - \hat{H}(m))S(m) * B(m) + D(m) - \hat{D}(m) \tag{8}$$

$\hat{H}(m)$ and $\hat{D}(m)$ are the estimated channel and the NLD respectively.

Ideally $\hat{H}(m) = H(m)$ and $\hat{D}(m) = D(m)$, therefore equation (7) becomes: $\hat{Y}(m) = W(m) + \eta X(m)$, which corresponds to an HD mode. Any mistake in the cancellation process may lead to a wrong decision about the PU presence.

3. The RSI effect on the Spectrum Sensing

In the spectrum sensing context, we usually assume two hypothesis: H_0 (PU signal is absent) and H_1 (otherwise). In our works, we assume that PU signal and SU signals are wideband signals such as OFDM. By focusing only on the additive receiver distortion which is dominated by the NLD of the LNA [2], the received signal can be modeled as follows:

$$Y_p(m) = H(m)S(m) + W(m) + D(m) + \eta X(m) \tag{9}$$

$H(m)$ is the channel between the SU transmitter antenna T_x and the SU receive antenna R_x, $S(m)$ is the SU signal, $W(m)$ is an Additive White Gaussian Noise (AWGN), $D(m)$ represents the NLD of the LNA, $X(m)$ is image of the the PU signal on R_x and $\eta \in \{0,1\}$ is the channel indicator ($\eta = 1$ if PU is active and $\eta = 0$ otherwise).

In order to decide the existence of the PU, several algorithms have been proposed in the literature [4]. The most commonly used one is the Energy Detector (ED), which compares the received signal energy, T_{ED}, to a predefine threshold, λ.

$$T_{ED} = \frac{1}{N}\sum_{m=1}^{N} |\hat{Y}(m)|^2 \underset{H_0}{\overset{H_1}{\gtrless}} \lambda \tag{10}$$

Contrary to HD, where Only the noise variance should be estimated prior to establishing the Spectrum Sensing, in FD the power of the RSI should be also taken into account. By assuming the $i.i.d$ property of $\epsilon(n)$ (the time domain version of $\xi(m)$), $w(n)$ and $x(n)$, then $\xi(m)$, $W(m)$ and $X(m)$ become $i.i.d.$ (See Appendix (A.1)). In this case, the distribution of T_{ED} should asymptotically follow a normal distribution for a large number of samples, N, according to the central limit theorem. Consequently, the probabilities of False Alarm, p^f_{fa}, and the Detection, p^f_d, under the FD mode can be obtained as follows (See Appendix (A.2)):

$$p^f_{fa} = Q(\frac{\lambda - \mu_0}{\sqrt{V_0}}) = Q\left(\frac{\lambda - (\sigma_w^2 + \sigma_d^2)}{\frac{1}{\sqrt{N}}(\sigma_w^2 + \sigma_d^2)}\right) \tag{11}$$

$$p^F_d = Q(\frac{\lambda - \mu_1}{\sqrt{V_1}}) = Q\left(\frac{\lambda - (\sigma_w^2 + \sigma_d^2 + \sigma_x^2)}{\frac{1}{\sqrt{N}}(\sigma_w^2 + \sigma_d^2 + \sigma_x^2)}\right) \tag{12}$$

Where $Q(\cdot)$ is the Q–function, and μ_i and V_i are the

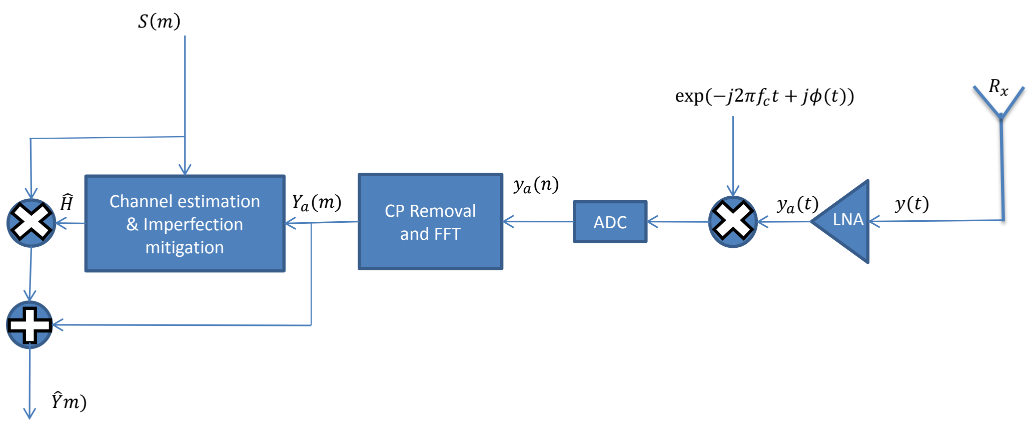

Figure 1. SIC circuit for OFDM receiver

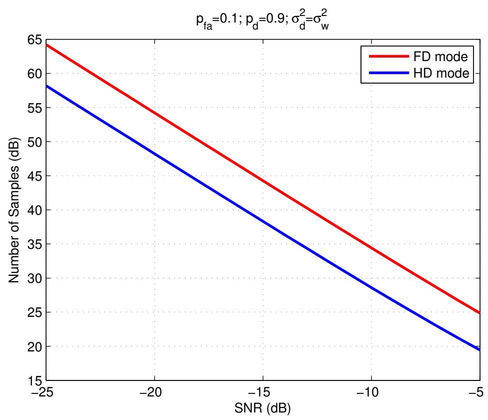

Figure 2. The number of samples required to reach $p_d = 0.9$ and $p_{fa} = 0.1$

mean and the variance of T_{ED} under H_i respectively, $i \in \{0; 1\}$, $\sigma_d^2 = E[|\xi(m)|^2]$ represents the RSI power, $\sigma_w^2 = E[|W(m)|^2]$ and $\sigma_x^2 = E[|X(m)|^2]$. The SNR, γ_x, is define as: $\gamma_x = \frac{\sigma_x^2}{\sigma_w^2}$. If the SIC is perfectly achieved, *i.e.* $\sigma_d^2 = 0$, p_{fa}^f and p_d^f take their expressions under the HD mode. Figure (2) shows the required number of samples to reach $p_d = 0.9$ and $p_{fa} = 0.1$ under the HD and FD modes for different values of SNR. In FD mode, we set $\sigma_d^2 = \sigma_w^2$ as the target values of σ_d^2 in digital communication. Figure (2) shows that the number of required samples slightly increases under the FD modes. For example, if $\gamma_x = -5$ dB, then 85 samples are enough to reach the target $(p_d; p_{fa})$ under the HD mode while under FD mode, around 300 samples are needed.

Let us defin the Probability of Detection Ratio (PDR), δ, for the same probability of false alarm under FD and HD modes, as follows:

$$\delta = \frac{p_d^f}{p_d^h} \; with \; p_{fa}^f = p_{fa}^h = \alpha \tag{13}$$

Where p_d^h and p_{fa}^h are the probabilities of detection and false alarm under HD respectively, $0 \leq \alpha \leq 1$ and $0 \leq \delta \leq 1$. As with an excellent SIC, the ROC can mostly reach in FD the same performance of HD. In order to show the effect of RSI on δ, let us defin the RSI to noise ratio γ_d as follows:

$$\gamma_d = \frac{\sigma_d^2}{\sigma_w^2} \tag{14}$$

Using (11) and (13), the threshold, λ, can be expressed as follows:

$$\lambda = \left(\frac{1}{\sqrt{N}} Q^{-1}(\alpha) + 1\right)(\sigma_w^2 + \sigma_d^2) \tag{15}$$

By replacing (15) in (12), γ_d can be expressed as follows:

$$\gamma_d = \frac{(1+\gamma_x)Q^{-1}(\delta p_d^h) - Q^{-1}(\alpha) + \sqrt{N}\gamma_x}{Q^{-1}(\alpha) - Q^{-1}(\delta p_d^h)} \tag{16}$$

If $\delta = 1$, then we can prove that γ_d becomes zero, which means that the SIC is perfectly achieved. Figure (3) shows the curves of γ_d for various values of PDR, δ, with respect to the SNR, γ_x, for $p_d^h = 0.9$ and $\alpha = 0.1$. This figur shows that as δ increases γ_d decreases. To

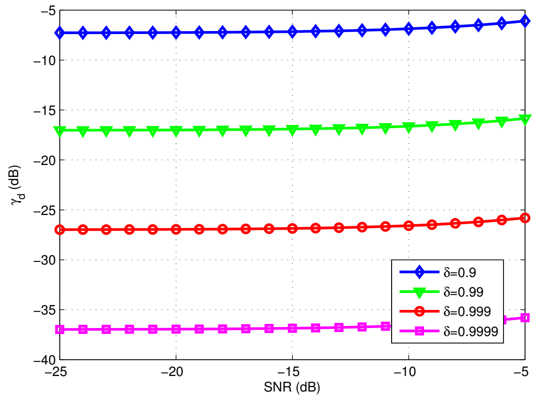

Figure 3. Evolution of γ_d with respect to γ_x for various values of δ, $(p^h_{fa}\ ;\ p^h_d) = (0.1\ ;\ 0.9)$

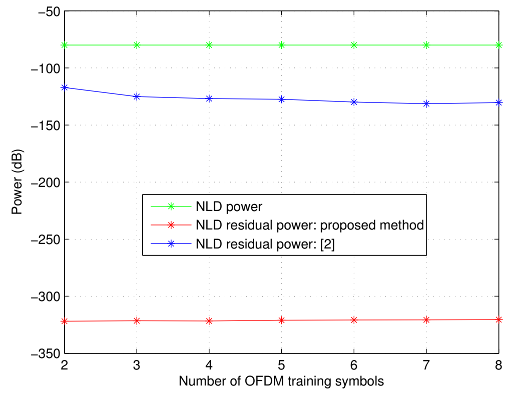

Figure 4. The effect of the number of training symbols on the NLD residual power

enhance the PDR, the selected SIC technique should mitigate at most the SI. In fact, for $\gamma_x = -5$ and a permitted loss of 1% (*i.e.* $\delta = 0.99$), γ_d is about -15 dB

4. The Non-Linear Distortion of the Low Noise Amplifier

In real world applications, the full duplex transceiver seems hard to be deployed due to hardware imperfections: the non-linearity of the amplifiers the quantization noise of ADC, the phase noise of the oscillator, *etc.* The NLD of LNA is an important performance limiting factor [2, 7–10]. According to NI 5791 datasheet [18], the NLD power is of 45 dB below the power of the linear amplifie component. A new efficient algorithm is proposed in this section, in order to mitigate as possible the NLD of LNA and to make the channel estimation more reliable. The proposed algorithms are analysed by neglecting the quantific tion and the phase noises.

4.1. Estimation of the Non-Linearity Distortion of LNA

The LNA output can be written as a polynomial of odd degrees of the input signal [17]. The NLD stands for the degrees greater than one. By limiting to the third degree and neglecting the higher degrees power [19], the NLD component can be written as follows:

$$d(t) = \beta y^3(t) \tag{17}$$

Where β is the NLD coefficient. The estimation of β can be helpful to suppress the LNA output. In this case, the channel estimation is no longer affected by the NLD. The overall output signal of the LNA, $y_a(t)$, can be expressed as follows:

$$y_a(t) = \theta y(t) + \beta y^3(t) \tag{18}$$

θ and $y(t)$ are the power gain and the input signal of the LNA respectively. To estimate θ and β, by a and b respectively, one can minimize the following cost function:

$$\mathcal{J} = E\left[\left(y_a(t) - (ay(t) + by^3(t))\right)^2\right] \tag{19}$$

By deriving $\mathcal{J}$ with respect to a and b we obtain:

$$\frac{\partial \mathcal{J}}{\partial a} = 0 \Rightarrow aE[y^2(t)] + bE[y^4(t)] = E[y_a(t)y(t)] \tag{20}$$

$$\frac{\partial \mathcal{J}}{\partial b} = 0 \Rightarrow aE[y^4(t)] + bE[y^6(t)] = E[y_a(t)y^3(t)] \tag{21}$$

Using equations (20) and (21), a linear system of equations can be obtained:

$$\begin{bmatrix} a \\ b \end{bmatrix} = A^{-1}B \tag{22}$$

Where:

$$A = \begin{bmatrix} E[y^2(t)] & E[y^4(t)] \\ E[y^4(t)] & E[y^6(t)] \end{bmatrix};\ B = \begin{bmatrix} E[y_a(t)y(t)] \\ E[y_a(t)y^3(t)] \end{bmatrix} \tag{23}$$

Once the non-linearity coefficient, β, is estimated, the non-linearity component can be subtracted from the output signal of the amplifie .

4.2. Numerical Results

Figure (4) shows the residual power of the NLD cancellation. The NLD power is fixe to 45 dB under the linear component [18]. This power is reduced to less than -300 dB after the application of our method. The method of [2] reduces the NLD power by around 50 dB. β is estimated using various number of training symbols, N_e. In this simulation, OFDM modulations are used with 64 sub-carriers and a CP length equal to 16. The received power is fixe to -5 dBm and the noise power to -72 dBm [18]. As shown in figur (4), the residual power of NLD decreases with an increasing of N_e when the method of [2] is applied. However our method keeps a constant value of this power. Our technique outperforms significa tly the method proposed in [2]. To show the impact of NLD on the channel estimation and the RSI power, figur (5) shows the power of $\hat{Y}(m)$ obtained in FD under H_0. The channel is estimated according to the method previousl y proposed by [20] as follows:

$$\hat{h} = IDFT\left\{\frac{1}{N_e}\sum_{k=0}^{N_e}\frac{Y_a^k(m)}{Y^k(m)}\right\} \; and \; \hat{H} = DFT\{\hat{h}(1,..,n_{tap})\} \tag{24}$$

Where $IDFT$ stands for the inverse discrete Fourier transform and n_{tap} is the channel order. The number of training symbols, N_e, is fixe to 4 symbols. The number of sub-carrier is 64, the transmitted signal is of -10 dB (*i.e.* 20 dBm), and the noise f oor is -102 dB (*i.e.* -72 dBm) [18]. The transceiver antenna is assumed to be omni-directional with 35 cm separation between T_x and R_x, so that a passive suppression of 25 dB is achieved [3]. According to the experimental results of [3], in a low reflectio environment, 2 channel taps are enough to perform the SIC when the passive suppression is bellow 45 dB. Furthermore, the line of sight channel is modelled as a Rician channel with K-factor about 20 dB. The non-line of sight component is modeled by a Rayleigh fading channel.

Figure (5) shows that our method leads to mitigate almost all the self interference, so that the power of $\hat{Y}(m)$ becomes very closed to the noise power. However, with the method of [2], the RSI power increases with the NLD power because the NLD power is a limiting factor of the channel estimation which leads to a bad estimation of the channel.

To show the impact of the NLD on the Spectrum Sensing, figur (6) shows the ROC in various situations under $\gamma_x = -10$ dB. The simulations parameters in this figur are similar to those of figur (5), only the NLD power is set to 45 dB under the linear component according NI 5791 indications [18]. The method of [2] leads to a linear ROC, which means that no meaningful information about the PU status can be obtained. By referring to figur (5), the RSI power is of -82 dB for a NLD power of -45 dBc, which means that γ_d in this case is about 20 dB. This high RSI power leads to a harmful loss of performance (see figur (3)). From the other hand, our method makes the ROC in FD mode almost colinear with that of the ROC of HD mode, which means that all SI and receiver impairments is mitigated.

Figure (7) shows the PDR for a target $\alpha = 0.1$ and $P_d^H = 0.9$. The ratio δ increases with the SNR. At a low SNR of -10 dB, δ becomes closed to 1, so that a negligible

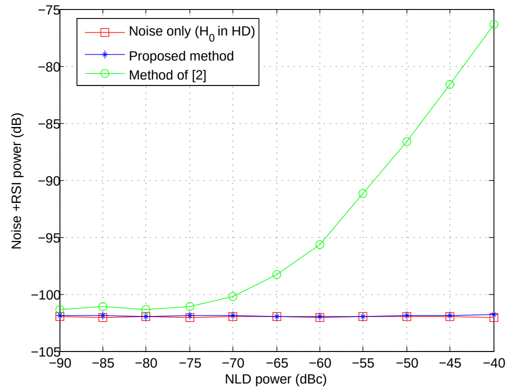

Figure 5. The power of $\hat{Y}(m)$ under H_0 obtained after applying: (1) our proposed method, (2) the method of [2] is applied and (3) under HD mode

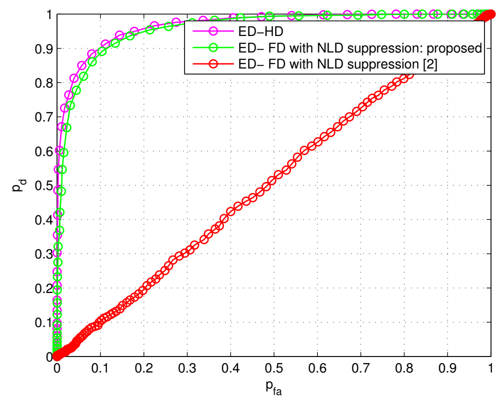

Figure 6. The ROC curve after applying the proposed technique of NLD suppression

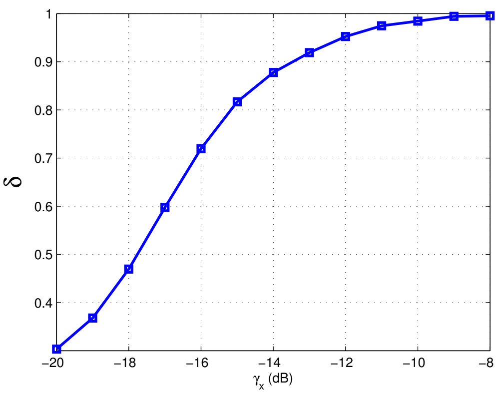

Figure 7. The gain ratio: $\delta = \frac{P_d^F}{P_d^H}$ for different values of SNR

performance loss is happen. As the SNR decreases the detection process in FD mode becomes more sensitive to the RSI power.

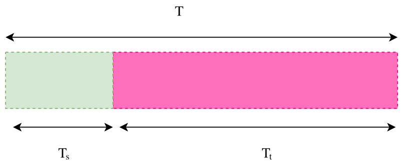

Figure 8. Activity period of SU under HD functioning is divide into Sensing sub-period (T_s) and Transmission sub-period (T_t).

5. Throughput Analysis

In this section, we analyse the SU performance by discussing from throughput viewpoint. Although FD-CR is introduced to enhance the SU data rate, the RSI affects both false alarm and detection probabilities as discussed in section (3). Consequently, the throughput of SU under FD can be lower than the throughput under HD for some circumstances.

First, let us present the activity period, T, of SU in HD mode (see figur (8)). We assume that this activity can be divided into sensing period (T_s) and transmitting period (T_t). The throughput of SU (R_{SU}) is affected by the sensing performance since the detection of the PU is not perfect and it can be done up to a target pair (p_{fa}, p_d). After the sensing period, SU can continue transmitting in one of the two following scenarios:

1. Missed Detection: SU decides that there is no active PU in the bandwidth, while PU is truly active. In this case, SU becomes active and interferes with PU

2. Reject: SU rejects the hypothesis H_1 by detecting correctly the absence of PU. Therefore SU becomes active

As SU is active under Missed Detection and Reject cases, R_{SU} becomes the sum of the two throughputs: R_0 under Reject case and R_1 under Missed Detection case.

$$R_{SU} = R_0 + R_1 \tag{25}$$

Let us denote by p_0 the probability that PU is truly absent, and p_1 the probability that PU is truly transmitting, with $p_0 + p_1 = 1$. Accordingly, R_1 and R_0 are given by [21]:

$$R_0 = \tau C_0\left(1 - p_{fa}\right)p_0 \tag{26}$$

$$R_1 = \tau C_1\left(1 - p_d\right)p_1 \tag{27}$$

where $\tau = \frac{T_t}{T}$ is the ratio of the transmission period with respect to the overall activity period, and C_0 (resp. C_1) is the throughput of SU when it operates under H_0 (resp. (H_1)). By assuming that SU and PU signals are statistically independent, white and Gaussian, C_0 and C_1 are given by:

$$C_0 = log_2(1 + \gamma_r) \tag{28}$$

$$C_1 = log_2(1 + \zeta_r) \tag{29}$$

where γ_r is the SNR and ζ_r and the Signal to Noise and Interference Ratio (SNIR) of SU signal at the Secondary receiver.

R_1 represents the interfering throughput of SU transmission to PU. This throughput should be minimized as possible by increasing p_d in order to do not affect the PU transmission. In contrast, R_0 represents the gainful throughput of SU to be maximized by decreasing p_{fa}, since it results from a true decision on the absence of PU. On the other hand, both R_0 and R_1 are related to τ. In FD-CR the ratio τ is equal to one since no silence period is required. Even though this fact is important for the SU throughput , it can affect the Spectrum Sensing reliability , by increasing p_{fa} (*i.e.* decreasing the gainful throughput R_0) and decreasing p_d (*i.e.* increasing the interference throughput R_1). To establish the minimum requirement of SU transmission in FD in order to exceed the throughput in HD; two conditions have to be respected:

1. the interfering throughput under HD mode represents the upper bound of the interfering throughput under FD mode. This means the interference amount under FD mode must not exceed that under HD mode.

2. The gainful throughput under HD mode represents the lower bound of the FD gainful throughput.

The firs condition can be expressed as follows:

$$C_1(1 - p_d^f)p_1 \leq \tau C_1(1 - p_d^h)p_1 \tag{30}$$

The left-hand side of the equation (30) represents the interfering throughput in FD mode, whereas the right-hand side represents the interfering throughput under HD mode. Accordingly, p_d^f becomes:

$$p_d^f \geq 1 - \tau(1 - p_d^h) \tag{31}$$

Equation (31) shows that p_d^f is a function of p_d^h and τ. The condition presented in equation (31) represents the limit detection rate to be reached ($p_d^f = 1 - \tau(1 - p_d^h)$) in order to respect the same interfering throughput of SU to PU transmission for given p_d^h and τ.

Figure (9) shows the evolution of p_d^f with p_d^h for various values of τ. Where τ grows, the limit of p_d^f decreases. This is due to the fact that in FD, SU does not stop transmitting, then the interfering

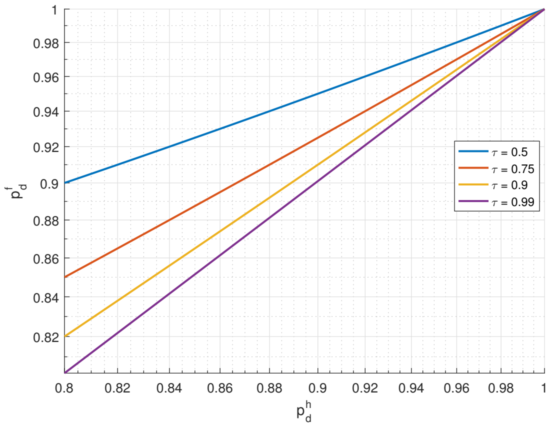

Figure 9. Variation of p^f_d in terms of p^h_d for various values of τ

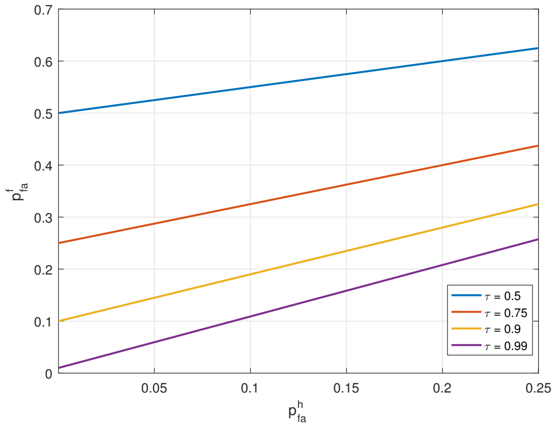

Figure 10. Variation of p^f_{fa} in terms of p^h_{fa} for various values of τ

throughput increases relatively to HD mode. Increasing the detection rate in FD comparing to HD leads to compensate the additional interference caused by the elimination of the silence period. For $\tau = 0.5$ and 0.75, p^f_d should be greater than 0.95 and 0.93 respectively for $p^h_d = 0.9$. For $\tau = 0.99$ (SU practically functions in FD mode), p^f_d and p^h_d becomes approximately of the same values.

Regarding the gainful throughput (2nd condition), R_0, the condition resulting in increasing the SU throughput in FD mode relatively to HD can be introduced by:

$$C_1(1 - p^f_{fa})p_1 \geq \tau C_1(1 - p^h_{fa})p_1 \quad (32)$$

This equation results in the upper bound of the false alarm rate of SU to be respected in order to increase the gainful throughput of FD relatively to HD:

$$p^f_{fa} \leq 1 - \tau(1 - p^h_{fa}) \quad (33)$$

Figure (10) shows the upper bound of p^f_{fa}, bellow which the FD gainful throughput exceeds the one of HD mode. p^h_{fa} and p^f_{fa} are linearly correlated. On the other hand, it is clear from this figur that the upper bound of p^f_{fa} grows inversely to τ. Increasing p_{fa} does not affect the PU transmission, but it prevents SU from using efficiently the spectrum opportunity .

In figur (11), we examine the gainful throughput (R_0) of SU in terms of SNR for various values of p_{fa}. Note that p^h_{fa} is fixe at 0.1, $\tau = 0.75$ and $p_0 = 0.9$. As shown in this figure the throughput decreases with the increase of p_{fa}, this is due to the spectrum opportunity loss. On the other hand, for $p^f_{fa} = 0.4$, the FD throughput becomes lower than the HD one. This means the functioning in HD mode becomes much interesting to SU.

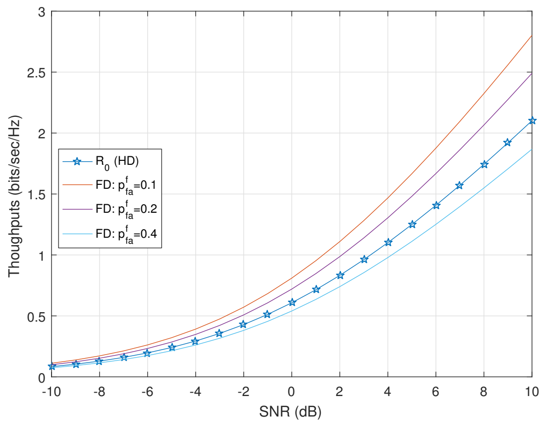

Figure 11. Evolution of the throughput of SU in terms of SNR (dB) for HD and FD mode

6. Conclusion

In this paper, the Spectrum Sensing and the Throughput of Full-Duplex Cognitive Radio are discussed. Regarding the Spectrum Sensing, a relation between the detection and false alarm probabilities under Full-Duplex and Half-Duplex modes is derived. Such a relation shows the effect of the Residual Self-Interference on the Spectrum Sensing performance. In order to reduce the RSI power, the Non-Linear Distortion (NLD) of the Low Noise Amplifie is addressed by proposing a new algorithm to suppress it. The proposed algorithm shows its efficiency by reducing the RSI power and leading to enhance the Spectrum Sensing performance. In addition, the Secondary User Throughput in Full-Duplex mode is analysed by deriving the upper bound of the false alarm rate and the lower bound of the detection rate. When these two bounds are not respected, the throughput of SU under Half-Duplex mode becomes higher than the one under Full-Duplex, which means no benefi is gained from the Full-Duplex functioning.

Appendix A. Appendix

A.1. *i.i.d.* property in Frequency Domain

Let $r(n)$ be an $i.i.d.$ time-domain signal. The DFT, $R(m)$, of the $r(n)$ is define as follows:

$$R(m) = \sum_{n=1}^{L} r(n)e^{-j2\pi m \frac{n}{L}} \quad (A.1)$$

Where L is the number of samples of $r(n)$. According to the Central Limit Theorem (CLT), $R(m)$ follows asymptotically a Gaussian distribution for a large L. Based on [22], two normal variables are independent if and only if (iff) they are uncorrelated.

The correlation, $C(m_1, m_2)$, of $R(m_1)$ and $R(m_2)$ $\forall$ $m_1 \neq m_2$ is given as follows:

$$\begin{aligned}
C(m_1, m_2) &= E[R(m_1)R^*(m_2)] \\
&= E\left[\sum_{n_1,n_2=1}^{L} r(n_1)r^*(n_2)e^{-j2\pi\frac{n_1 m_1 - m_2 n_2}{L}}\right] \\
&= \sum_{n_1=n_2=1}^{L} E\left[|r(n_1)|^2\right] e^{-j2\pi\frac{(m_1-m_2)n_1}{L}} \\
&\quad + \sum_{n_1\neq n_2=1}^{L} \underbrace{E\left[r(n_1)r^*(n_2)\right]}_{=0,\ since\ r(n)\ is\ i.i.d.} e^{-j2\pi\frac{(n_1 m_1 - n_2 m_2)}{L}} \\
&= E\left[|r(n_1)|^2\right] \underbrace{\sum_{n_1=1}^{L} e^{-j2\pi(m_1-m_2)\frac{n_1}{L}}}_{=0} = 0 \quad (A.2)
\end{aligned}$$

As $C(m_1, m_2) = 0$; $\forall$ $m_1 \neq m_2$, therefore $R(m_1)$ and $R(m_2)$ become uncorrelated and independent since they are Gaussian.

A.2. Probability of Detection and Probability of False Alarm

As by our assumption $\xi(m)$, $W(m)$ and $X(m)$, are asymptotically Gaussian $i.i.d.$, then $\hat{Y}(m)$ is also Gaussian and $i.i.d.$ Therefore, the Test Statistic, T_{ED}, of equation (10) follows a normal distribution according to CLT for a large N. Under H_0 ($i.e.$ $X(m)$ does not exist), the mean, μ_0, and the variance, V_0 of T_{ED} can be obtained as follows:

$$\mu_0 = E[T] = E\left[\frac{1}{N}\sum_{m=1}^{N} |\xi(m) + W(m)|^2\right] = \sigma_w^2 + \sigma_d^2 \quad (A.3)$$

$$\begin{aligned}
V_0 &= E[T^2] - E^2[T] = \frac{1}{N^2}E\left[\left(\sum_{m=1}^{N}|\hat{Y}(m)|^2\right)^2\right] - (\sigma_w^2 + \sigma_d^2)^2 \\
&= \frac{1}{N^2}E\left[\sum_{m_1=m_2=1}^{N} |\hat{Y}(m_1)|^4\right] \\
&\quad + \frac{1}{N^2}E\left[\sum_{m_1\neq m_2=1}^{N} |\hat{Y}(m_1)|^2\hat{Y}(m_2)|^2\right] - (\sigma_w^2 + \sigma_d^2)^2 \\
&= \frac{1}{N^2}\sum_{m_1=m_2=1}^{N} E\left[|\hat{Y}(m_1)|^4\right] - \frac{1}{N}(\sigma_w^2 + \sigma_d^2)^2 \quad (A.4)
\end{aligned}$$

Since $\hat{Y}(m)$ is Gaussian, then its kurtosis $kurt(\hat{Y}(m))$ is zero.

$$kurt(\hat{Y}(m)) = E[|\hat{Y}(m)|^4] - E[\hat{Y}^2(m)] - 2E^2[|\hat{Y}(m)|^2] = 0 \quad (A.5)$$

Assuming that the real and the imaginary parts of $\hat{Y}(m)$ are independent and of the same variance then $E[\hat{Y}^2(m)]$ becomes zero. Therefore: $E[|\hat{Y}(m)|^4] = 2E^2[|\hat{Y}(m)|]^2 = 2(\sigma_w^2 + \sigma_d^2)^2$. Back to equation (A.4), the variance, V_0 becomes:

$$V_0 = \frac{1}{N}(\sigma_w^2 + \sigma_d^2)^2 \quad (A.6)$$

By following the same procedure, μ_1 and V_1 can be obtained as follows under H_1 ($X(m)$ exists):

$$\mu_1 = \sigma_w^2 + \sigma_d^2 + \sigma_x^2 \quad (A.7)$$

$$V_1 = \frac{1}{N}(\sigma_w^2 + \sigma_d^2 + \sigma_x^2)^2 \quad (A.8)$$

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