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# ON THE RECURRENCE PLOT ANALYSIS METHOD BEHAVIOUR UNDER SCALING TRANSFORM

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## ABSTRACT

In the last decade, the applications of the recurrence plot analysis method make it a valuable alternative to the time-frequency and time-scale tools. As it was initially developed for the study of dynamical systems, and was later used in nonlinear time series analysis, the question of using it as a signal processing tool has not been put into discussion yet. In this field the projective techniques are largely used, with good results. Nevertheless, they also have some limitations – especially regarding transient signal processing. But this kind of signals are ubiquitous in real world. In addition, propagation through various media as well as on multiple paths lead to delayed, attenuated and dilated versions of the original transients. In this paper we study the behaviour of the recurrence plot analysis method in the context of analyzing some finite duration signals being subject to rescalings of the amplitude and time axes. This study is a starting point in employing the analysis of recurrences in investigations of a large class of real world signals.

**Index Terms**— Transient signals, recurrence plots, amplitude and time scaling

## 1. INTRODUCTION

Recurrence plot analysis [1] has its roots in a 1987 paper by Eckman et al [2]. They introduced the recurrence plot as a "new graphical tool for measuring the time constancy of dynamical systems". Although its initial purpose was to obtain dynamical parameters from time series, it was subsequently applied in various areas of the field of nonlinear time series analysis. According to [3], the methods based on recurrence plots have been successfully applied in "physiology, neuroscience and genomics, ecology, physics, chemistry, earth science and astrophysics, engineering and economy". The bibliography gathered on the recurrence plot website (<http://www.recurrence-plot.tk>) shows significant increasing of interest for this tool in the last ten years, with a peak in the last three years. However, very few applications of recurrence plots were reported in signal processing – mostly in speech processing and signal detection (e.g. [4, 5, 6, 7]). The integration of this technique in the signal processing field seems natural, as there is a strong similarity between recurrence and frequency. This represents the main objective of our paper. Hence, we start by studying the effects of scaling the amplitude and the time axis of the signal on the resulting recurrence plot. Scaling phenomena (like attenuations and dilatations) are common in real world signals. The issue of dilatations has been solved by the wavelet transform, but the choice of the proper mother

wavelet is not always possible for all signals. On the other hand, recurrence plot analysis does not need a reference signal, as it only compares the analyzed signal to itself.

The paper is organized as follows. In the following, we introduce first the principle of recurrence plot analysis and we continue by presenting the effects of scaling the signal amplitude. Then, we present the effects of time scaling and we continue with a short discussion regarding the potential of the obtained results. We close by pointing out the conclusions.

## 2. RECURRENCE PLOT ANALYSIS

Figure 1 summarizes the generic steps of the recurrence plot analysis method. They are briefly discussed in the following paragraphs.

(1) – Representation of the signal as a trajectory in a multidimensional space. This is performed with the aid of the method of delays. If  $m$  is the dimension of the representation space and  $\tau$  is the time delay, then the phase space trajectory corresponding to the signal  $s(t)$  is:

$$\vec{r}(t) = \sum_{k=1}^m s(t + (k-1)\tau) \cdot \vec{e}_k, \quad (1)$$

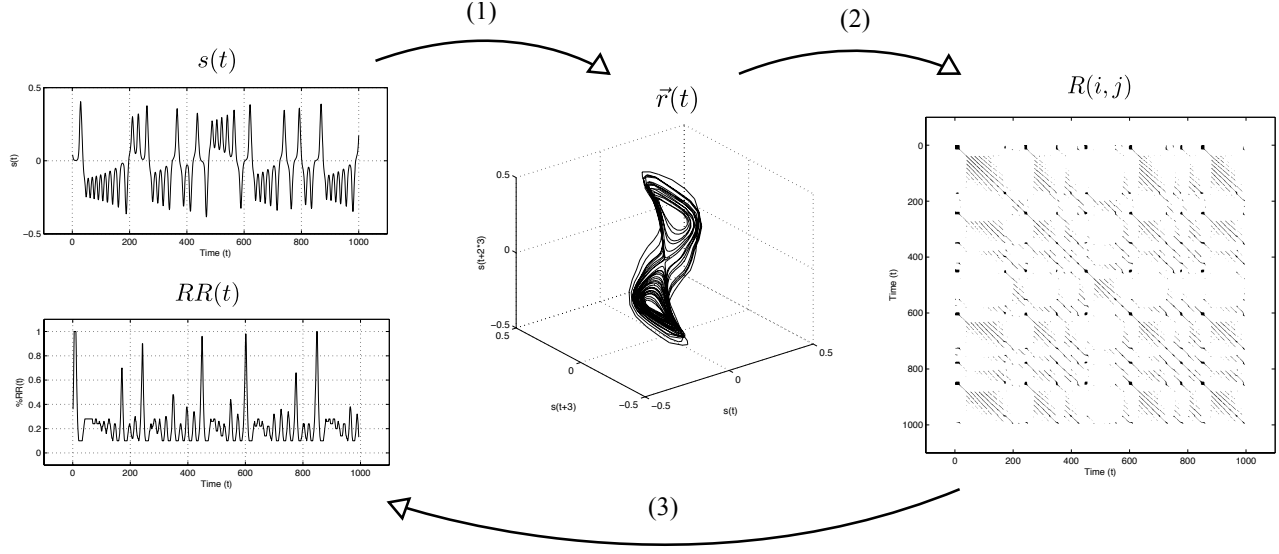
where  $\vec{e}_k$  are the versors of the phase space axes. Hence, the time evolution of the trajectory will be the same on all the  $m$  axes, except for time delayings by multiples of  $\tau$ .

(2) – Computation of the recurrence plot. A recurrence between points at times  $i$  and  $j$  on the trajectory is defined as:

$$R(i, j) = \Theta(\varepsilon(i) - D(\vec{r}(i), \vec{r}(j))), \quad (2)$$

where  $\Theta$  denotes the Heaviside step function, and  $D(\vec{r}(i), \vec{r}(j))$  stands for the distance between points  $i$  and  $j$  on the trajectory. In fact, Equation (2) shows that a recurrence is identified between points  $i$  and  $j$  on the trajectory if the distance between them is smaller than a threshold  $\varepsilon(i)$ , known as the recurrence radius. Most often, a constant  $\varepsilon$  is used, and the distance  $D$  is computed using the Euclidean metric. (We noticed that in most cases it is sufficient to choose  $\varepsilon$  as the mean distance between successive points of the trajectory.) However, it might sometimes be useful to work directly with the distance plot (i.e the graphical representation of  $D$ ).

(3) – Quantification of the recurrence plot. It consists in performing different computations on the recurrence plot, in order to obtain some measures that are able to offer some insight into the analyzed signal. The most common such measure is the recurrence



**Fig. 1.** The steps of recurrence plot analysis: (1) phase space representation of the signal; (2) computation of the recurrence plot; (3) quantification of the recurrence plot. In this example, signal  $s(t)$  is obtained from a chaotical Lorenz system. The trajectory  $\vec{r}(t)$  is obtained using a dimension  $m = 3$  and a delay  $\tau = 3$ . It can be noticed that  $\vec{r}(t)$  has strong similarities with the well-known Lorenz "butterfly" attractor. The transitions of the system from one unstable periodic orbit to the other can be identified using the significant maxima of the locally-computed (by using a window of size 10) recurrence rate,  $RR(t)$ .

rate,  $RR$ . It is computed as the ratio between the total area occupied by black dots (recurrences) in the recurrence plot and the total area of the recurrence plot. The recurrence rate, as well as other quantification measures, can be computed either globally, for the whole recurrence plot, or locally, on time windows that are being shifted along the main diagonal of the recurrence plot. In this way, a time-varying measure,  $RR(t)$ , will be obtained.

Various recommendations can be found in literature regarding the choice for the parameters of the recurrence plot analysis method (i.e.  $m, \tau, \varepsilon, D$ ) [3]. Only the most commonly used recurrence-based analysis method was presented in this section.

### 3. AMPLITUDE SCALING

As the phase space representation of the signal is a linear operation, scaling the signal amplitude will result in an appropriate scaling of the trajectory. If for signal  $s(t)$  we have trajectory  $\vec{r}(t)$ , then for signal  $s'(t) = \alpha \cdot s(t)$  we have:

$$\vec{r}'(t) = \sum_{k=1}^m \alpha \cdot s(t + (k-1)\tau) \cdot \vec{e}_k = \alpha \cdot \vec{r}(t). \quad (3)$$

Let us see whether this scaling of the trajectory has any effect on the recurrence plot. If the recurrence plot corresponding to signal  $s(t)$  is  $R(i, j)$ , then the recurrence plot corresponding to signal  $s'(t)$  will be:

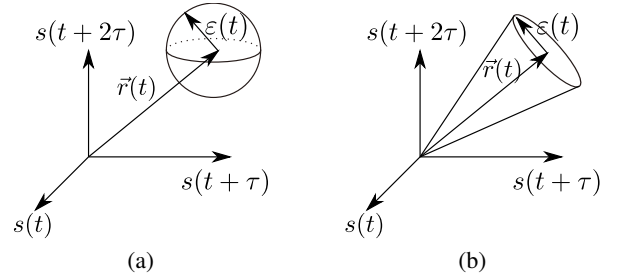
$$R'(i, j) = \Theta(\varepsilon'(i) - D(\alpha \cdot \vec{r}(i), \alpha \cdot \vec{r}(j))). \quad (4)$$

The distance  $D$  is usually computed using a certain metric, that is:

$$D(\vec{r}(i), \vec{r}(j)) = \|\vec{r}(i) - \vec{r}(j)\|. \quad (5)$$

In this case, in order for  $R'(i, j)$  to be identical to  $R(i, j)$ ,  $\varepsilon'(i)$  should be chosen such that:

$$\varepsilon'(i) = \alpha \cdot \varepsilon(i). \quad (6)$$

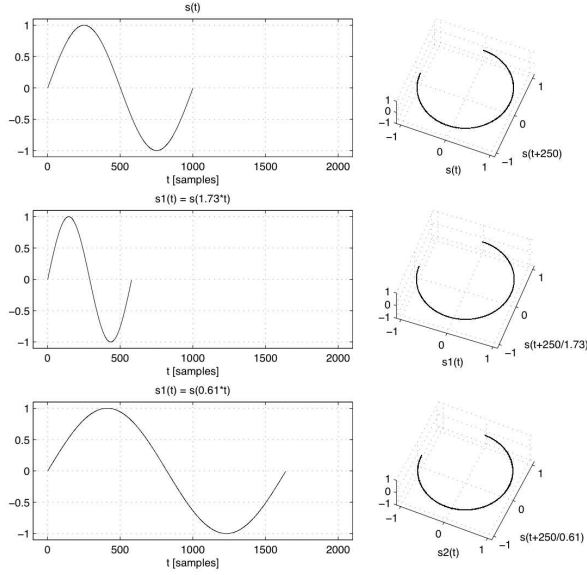


**Fig. 2.** The shape of the neighbourhood (in a tridimensional phase space) is a sphere for the Euclidean metric (a) and a cone for the angular distance (b).

However, this scaling of the recurrence radius is no more needed when using an angular distance, defined as:

$$D(\vec{r}(i), \vec{r}(j)) = \arccos\left(\frac{\vec{r}(i) \cdot \vec{r}(j)}{\|\vec{r}(i)\| \cdot \|\vec{r}(j)\|}\right), \quad (7)$$

for non-zero vectors (otherwise vectors can be considered parallel, i.e. the angular distance between them is 0). This definition for  $D$  leads to a recurrence plot that is invariant to amplitude scalings of the analyzed signal. We note that this angular distance generates conical neighbourhoods (instead of the spherical ones generated by the Euclidean metric). Figure 2 illustrates this observation. We also note that by using such a conical neighbourhood the concept of recurrence loses its initial meaning (that is returning close to a previously visited point) – therefore, we are actually working with a generalized recurrence.



**Fig. 3.** Time-scaled one-period sinusoidal signal and the corresponding trajectories. From top to bottom,  $\beta$  is: 1, 1.73, 0.61, and  $\tau$  is: 250, 250/1.73, 250/0.61. The shape of the trajectory remains unchanged.

#### 4. TIME SCALING

If  $s(t)$  is the signal, the time-scaled version of it is  $s'(t) = s(\beta \cdot t)$ . The corresponding trajectory will be:

$$\vec{r}'(t) = \sum_{k=1}^m s(\beta t + (k-1)\beta\tau') \cdot \vec{e}_k. \quad (8)$$

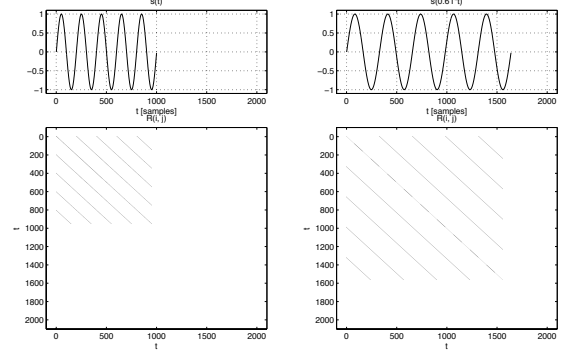
We notice that scaling the time axis of the signal leads to an appropriate scaling of the time delay (the  $\tau$  parameter). For the trajectory to remain unchanged,  $\tau'$  should be chosen such that:

$$\tau' = \frac{\tau}{\beta}. \quad (9)$$

Figure 3 shows that the trajectories indeed remain unchanged when  $\tau$  is properly scaled. However, Equation (8) shows that  $\vec{r}'(t)$  is not identical to  $\vec{r}(t)$ . Figure 3 does not reflect the time evolution, and therefore  $\vec{r}'(t)$  and  $\vec{r}(t)$  seem identical. In fact, trajectory  $\vec{r}'(t)$  is "drawn"  $\beta$  times faster (or slower, depending on whether  $\beta$  is greater than or less than 1) than trajectory  $\vec{r}(t)$ . This leads to the contraction/dilatation of the structures in the recurrence plot, as shown in Figure 4.

#### 5. DISCUSSION

We have shown in section 3 that we can obtain a recurrence plot that is invariant to amplitude scaling by using the angular distance. As for the time scaling, we showed in the previous section that by appropriately scaling the time delay the shape of the trajectory remains unchanged (Figure 3), except it is covered with a different speed, which results in a stretching of the recurrence plot pattern (Figure 4). Failure to meet the constraint in Equation (9), though, leads to



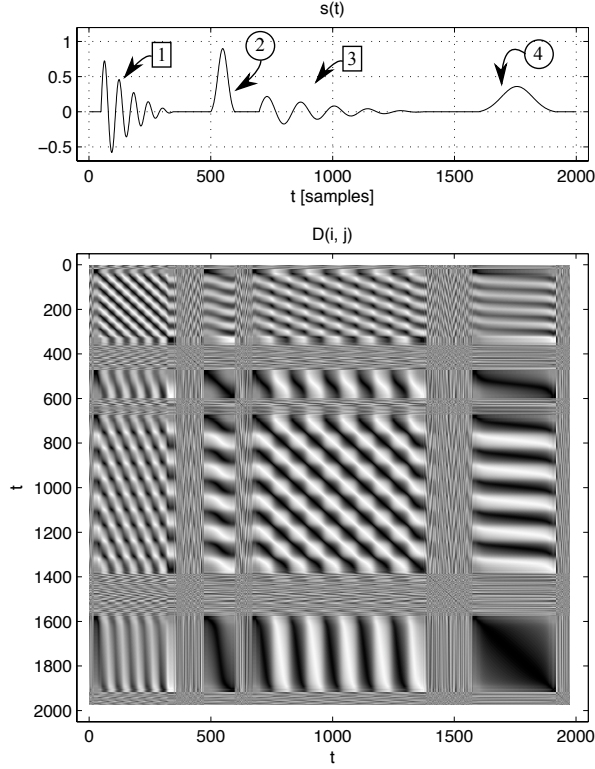
**Fig. 4.** The effect of dilatation on the recurrence plot, for a sinusoidal signal. (left) The recurrence plot of signal  $s(t)$  is obtained using  $m = 2$  and  $\tau = 50$ . (right) The recurrence plot of signal  $s(0.61 \cdot t)$  is obtained using  $m = 2$  and  $\tau' = 82 (= \tau/0.61)$ . As  $\tau$  is scaled appropriately, the pattern remains unchanged, except for the fact that it is stretched.

significant changes in the trajectory – redundancy (when  $\beta < 1$ ) or irrelevance (when  $\beta > 1$ ) [8]. It follows that the recurrence patterns will also be modified. However, when dealing with dilated signals, i.e. when  $\beta < 1$ , the trajectory does not change its main characteristics, except that it is "attracted" to the first diagonal of the coordinate system. (In this case the changes in the recurrence pattern are mostly due to the use of recurrence balls that have the same dimensions as those used for the non-dilated signal.)

Figure 5 shows an example of an angular distance plot computed for a synthetic signal (and Figure 6 repeats the example, after adding some noise). The signal is composed of four transients coming from two different sources – the first two of them come from direct propagation between the sources and the receiver, and the last two come from reflexions (i.e. they are attenuated and dilated). As the figure shows, the angular distance plot of this signal allows a good time localization of the four transients, regardless of their scale differences. More than that, it allows a visual characterization of the transients. It can be easily observed (by analyzing the blocks on the main diagonal, that correspond to the four transients) that we are dealing with two types of signals (i.e. two different sources). It can also be easily observed that the first transient is of the same type as the third, and that the second is of the same type as the fourth. We can make this statement either as a result of a comparative visual analysis of their corresponding blocks in the angular distance plot, or as a result of the fact that their cross-term blocks are composed of diagonally oriented structures (e.g. the third rectangular block on the first line of  $D(i, j)$  in Figure 5 is the cross-term block between the first and the third transients; all the black structures in it are oriented along the first diagonal of the block). This final observation may offer, together with the observations made in section 4, a good starting point for developing a robust automatic method for comparing transient signals.

#### 6. CONCLUSION

We presented in this paper the essential aspects of a nonlinear time series analysis method called recurrence plot analysis. As this tool is working its way into signal processing, we proposed to study its properties in the context of modifying the amplitude scale as well as



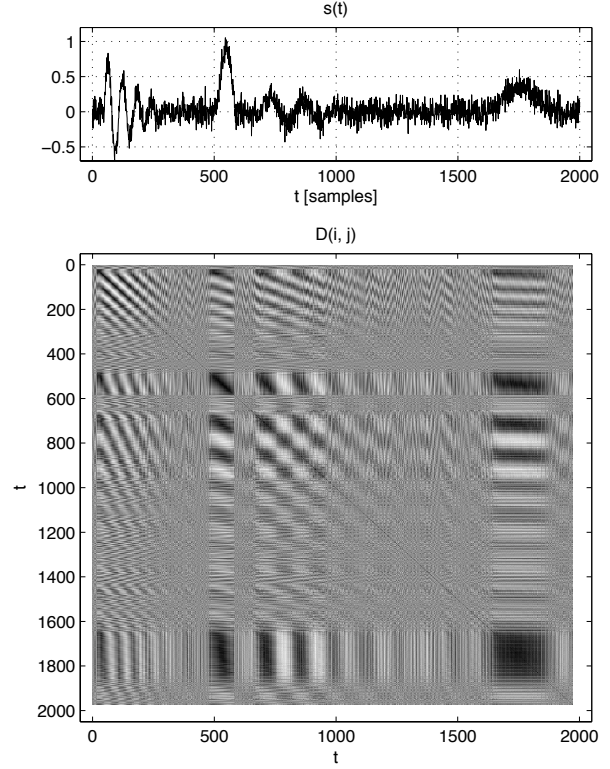
**Fig. 5.** Angular distance plot for a synthetic signal composed of four transients coming from two different sources. (The signal-to-noise ratio is 60 dB.) It is computed using  $m = 3$  and  $\tau = 15$ . The four transients can be easily localized in time and characterized as well by a visual analysis of this image.

the time scale of the signal. We showed that an amplitude scaling does not lead to a different recurrence plot as long as the recurrence radius is scaled appropriately. Besides that, we showed an alternative method to compute the recurrence plot, by using the angular distance between vectors, and we showed that the recurrence plot obtained this way is invariant to signal amplitude scaling. We also showed that a time scaling of the signal does not lead to a modified trajectory if the time delay is appropriately scaled. The recurrence plot remains also unchanged, except for the fact that it will compact/expand (without otherwise affecting the characteristics of the patterns it contains).

This theoretical study may serve as a basis for involving recurrence plot analysis in solving problems in the analysis of transient signals, that are encountered in many real world applications, where attenuations and dilatations are very common. Further, we are planning to use the observations made in this paper for developing a scale-independent method for the characterization of transient signals.

## 7. REFERENCES

- [1] N. Marwan, “A historical review of recurrence plots,” *The European Physical Journal Special Topics*, vol. 164, pp. 3–12, 2008.
- [2] J.-P. Eckmann, S. Oliffson Kamhorst, and D. Ruelle, “Recur-



**Fig. 6.** Angular distance plot for the same signal in Figure 5, with a signal-to-noise ratio of 5 dB. It is computed using the same parameters ( $m = 3$  and  $\tau = 15$ ). Noise has a strong impact on the resulted image, but time localization and characterization of the four transients can still be done to some extent.

- rence plots of dynamical systems,” *Europhysics Letters*, vol. 4, no. 9, pp. 973–977, November 1987.
- [3] N. Marwan, M. Carmen Romano, M. Thiel, and J. Kurths, “Recurrence plots for the analysis of complex systems,” *Physics Reports*, vol. 438, pp. 237–329, January 2007.
- [4] S. Schinkel, O. Dimigen, and N. Marwan, “Selection of recurrence threshold for signal detection,” *The European Physical Journal Special Topics*, vol. 164, pp. 45–53, 2008.
- [5] J. V. Michalowicz, J.M. Nichols, and F. Bucholtz, “Signal detection based on recurrence matrix statistics,” *Physics Letters A*, vol. 372, pp. 7172–7178, 2008.
- [6] G. K. Rohde, J. M. Nichols, B. M. Dissinger, and F. Bucholtz, “Stochastic analysis of recurrence plots with applications to the detection of deterministic signals,” *Physica D*, vol. 237, pp. 619–629, 2008.
- [7] N. Marwan and J. Kurths, “Comment on ”stochastic analysis of recurrence plots with applications to the detection of deterministic signals” by rohde et al. [physica d 237 (2008) 619-629],” *Physica D*, vol. 238, pp. 1711–1715, 2009.
- [8] N. Marwan, *Encounters with neighbours. Current developments of concepts based on recurrence plots and their applications*, Ph.D. thesis, Institut für Physik, Fakultät Mathematik und Naturwissenschaften, Universität Potsdam, May 2003.